

A New Proposal for the Source of Curvature in Einstein's Vacuum Solutions

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ABSTRACT: This paper analyses new proposal which provides that it is always possible to define the source of curvature in a vacuum solution of Einstein's field equations in terms of some dimensional parameters. As the parameters vanish so does the curvature. It also helps to make the vacuum solutions Machian.

KEYWORDS: Curvature, Einstein's field equations, Kretschmann scalar, Minkowskian form.

I. INTRODUCTION

It is well known that gravitation is a manifestation of the curvature of space-time with matter as the source of curvature. Now on the same line one should expect some source of curvature in the vacuum solution of Einstein field equations

$$R^{ik} = 0 \quad (1)$$

The source of curvature in vacuum solution is defined by a singularity appearing in solution of eq. (1) or in terms of the divergence of the Kretschmann scalar

$$K = R_{iklm} R^{iklm} \quad (2)$$

where R_{iklm} be the Riemann tensor. This view of source in terms of singularity does not work universally because there are several solutions of equations (1) which are curved but singularity free. Hence, it is an important to define the presence of source which may be applied universally to all the solutions of the equations (1).

II. BASIC FIELD EQUATIONS AND SOLUTIONS

Let us explore the nature of hidden source of curvature in the solutions of eq. (1) in well known solutions.

2.1 Schwarzschild Solution:

K. Schwarzschild (1915) investigated an exact solution of a static spacetime structure outside a static isotropic mass M kept in an empty space, and which reads

$$ds^2 = \left(1 + \frac{N}{r}\right) c^2 dt^2 - \left(1 + \frac{N}{r}\right)^{-1} dr^2 - r^2 d^2\theta - r^2 \sin^2 \theta d\varphi^2, \quad (3)$$

where N be a constant parameter as a constant of integration. But for a weak gravitation field one obtains

$$N = - \frac{2 G M}{c^2}. \quad (4)$$

The Kretschmann scalar reads

$$K = \frac{12 N^2}{r^6} \quad (5)$$

and generally it determines the source of curvature. It is obvious that the parameter N contains the source. For $N = 0$, the solution (3) reduces to the Minkowskian form. Hence, without invoking the singularity, it is possible to define the source in solution (3) by the parameter N.

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2.2 Kerr Solution

Kerr (1963) discovered solution for the mass M rotating as

$$ds^2 = \left(1 + \frac{Nr}{r^2 + \alpha^2 \cos^2 \theta}\right) c^2 dt^2 - \left(\frac{r^2 + \alpha^2 \cos^2 \theta}{r^2 + Nr + \alpha^2}\right) dr^2 - (r^2 + \alpha^2 \cos^2 \theta) d\theta^2 - \left(r^2 + \alpha^2 - \frac{\alpha Nr}{r^2 + \alpha^2 \cos^2 \theta} \sin^2 \theta\right) \times \sin^2 \theta d\phi^2 - \left(\frac{2\alpha Nr}{r^2 + \alpha^2 \cos^2 \theta}\right) \sin^2 \theta d\phi c dt, \quad (6)$$

With the Kretschmann scalar as

$$K = 12 N^2 \frac{[r^6 - \alpha^2 \cos^2 \theta (15r^4 - 15\alpha^2 r^2 \cos^2 \theta + \alpha^4 \cos^4 \theta)]}{(r^2 + \alpha^2 \cos^2 \theta)^6}, \quad (7)$$

Where α as the angular momentum per unit mass with total angular momentum = $M c \alpha$, with presence of singularity at $r = 0$ and $\theta = \pi/2$.

For $\alpha = 0$, it reduces to the Schwarzschild solution. Hence, without invoking singularity, it is possible to define the source of curvature to the parameters N and α , which come in Riemann tensor. Again vanishing of Riemann tensor it reduces to Minkowskian form for N and $\alpha = 0$. Hence, α and N are source carrier parameters. This indicates that the angular momentum also contributes to the curvature.

2.3 Taub-NUT solution:

Taub (1951) discovered another generalization of the Schwarzschild solution and Newman, Unit and Tamburino (1963) rediscovered. The solution reads,

$$ds^2 = \left(\frac{r^2 + Nr - L^2}{r^2 + L^2}\right) (C dt + 2L \cos \theta d\phi)^2 - \left(\frac{r^2 + L^2}{r^2 + Nr - L^2}\right) dr^2 - (r^2 + L^2) (d\theta^2 + \sin^2 \theta d\phi^2), \quad (8)$$

Which reduces to the Schwarzschild solution for $L = 0$. The solution is singularity free and regular at $r = 0$ as given by Kretschmann scalar

$$K = \frac{12}{(L^2 + r^2)^6} [N^2 r^6 + L^2 (4L^6 - N^2 L^4 - 24NL^4 r - 60L^4 r^2) + 15N^2 L^2 r^2 + 80NL^2 r^3 - 15N^2 r^4 + 60L^2 r^4 - 24Nr^5 - 4r^6] \quad (9)$$

But it has wire singularities at $\theta = 0$ and $\theta = \pi$ where the line element fails to be invertible. If one defines the source in terms of the source carrier parameters N and L which appear in the Riemann tensor. The parameter N in terms of mass of the source and parameter L in terms of the momentum P of the source. Hence, one obtains

$$L = K_1 \frac{GP}{c^3}. \quad (K_1 \text{ as the dimensionless number}) \quad (10)$$

Again for $N = 0$, the solution becomes Minkowskian.

2.4 Kasner Solution:

Kasner (1921) obtained the solution as

$$ds^2 = c^2 dt^2 - T^{2p_1} dx^2 - T^{2p_2} dy^2 - T^{2p_3} dz^2, \quad (11)$$

Where p_1, p_2 and p_3 being dimensionless parameter satisfy

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$$p_1 + p_2 + p_3 = 1 = p_1^2 + p_2^2 + p_3^2. \quad (12)$$

The solution (11) has singularity at $T = 0$ for which Kretschmann scalar reads

$$K = \frac{16(1 - p_3)p_3^2}{T^4}. \quad (13)$$

However, there is no any dimensional parameter which may be associated to source of curvature. Kasner solution was rediscovered by Narlikar and Karmarkar (1946) with following transformation

$$\left. \begin{aligned} T &= \frac{1 + nt}{n}, \\ X &= n^{p_1} x, \\ Y &= n^{p_2} y, \\ Z &= n^{p_3} z, \end{aligned} \right\} \quad (14)$$

Hence, one obtains

$$ds^2 = c^2 dt^2 - (1 + nt)^{2p_1} dx^2 - (1 + nt)^{2p_2} dy^2 - (1 + nt)^{2p_3} dz^2, \quad (15)$$

Where n as an arbitrary constant parameter. One may evaluate non-vanishing components of Riemann tensor, which is zero for $n = 0$. A simple dimensional analysis shows that n may be related to momentum density Q , as

$$n = K_2 \sqrt{\frac{GQ}{C}} \quad (\text{where } K_2 \text{ as the dimensionless number}) \quad (16)$$

Again K for eq. (15) reads

$$K = \frac{16n^4(1 - p_3)p_3^2}{(1 + nt)^4}. \quad (17)$$

But for $Q = 0$, it reduces to Minkowskian form. Hence, the source of curvature may be defined in terms of momentum density Q .

III. CONCLUSION

We have interpreted the source of curvature in vacuum solutions in terms of dimensional parameters in different known vacuum solutions.

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